宿題１０　　解答例

問１．授業で示した (4), (5), (6) 式に偏微分の連鎖則を適応して極座標の∂ / ∂x , ∂ / ∂y , ∂ / ∂z を求め、もう一度偏微分の連鎖規則を適応して $\frac{\partial^2}{\partial x^2}$, $\frac{\partial^2}{\partial y^2}$, $\frac{\partial^2}{\partial z^2}$ を求めた後ラプラシアを求めよ。
ホームページの lesson10a.pdf を参照して下さい。

問２．角運動量の各成分 Lx , Ly , Lz を極座標表示にせよ。
　　問１で得られた結果を使って

$$L_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right) = -i\hbar\left\{r\sin\theta\sin\phi\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\right.$$
$$\left. - r\cos\theta\left(\sin\theta\sin\phi\frac{\partial}{\partial r} + \frac{\cos\theta\sin\phi}{r}\frac{\partial}{\partial\theta} + \frac{\cos\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right)\right\}$$
$$= -i\hbar\left\{-\sin\phi\frac{\partial}{\partial\theta} - \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right\}$$

$$L_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) = -i\hbar\left\{r\cos\theta\left(\sin\theta\cos\phi\frac{\partial}{\partial r}\right.\right.$$
$$\left. + \frac{\cos\theta\cos\phi}{r}\frac{\partial}{\partial\theta} - \frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right) - r\sin\theta\cos\phi\left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\Big\}$$
$$= -i\hbar\left(\cos\phi\frac{\partial}{\partial\theta} - \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right)$$

$$L_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = -i\hbar\left\{r\sin\theta\cos\phi\left(\sin\theta\sin\phi\frac{\partial}{\partial r}\right.\right.$$
$$\left. + \frac{\cos\theta\sin\phi}{r}\frac{\partial}{\partial\theta} + \frac{\cos\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right) - r\sin\theta\sin\phi\left(\sin\theta\cos\phi\frac{\partial}{\partial r}\right.$$
$$\left.\left. + \frac{\cos\theta\cos\phi}{r}\frac{\partial}{\partial\theta} - \frac{\sin\phi}{r\sin\theta}\frac{\partial}{\partial\phi}\right)\right\} = -i\hbar\frac{\partial}{\partial\phi}$$